Independent Composition

Dependent Composition $P.Q$ (sequential execution)

P and Q must have exactly the same state variables

Independent Composition $P||Q$ (parallel execution)

P and Q must have completely different state variables

and the state variables of the composition are those of both P and Q

Ignoring time and space variables

$$P||Q \;=\; P\land Q$$

Independent Composition

example in integer variables x , y , and z

$x := x+1 \parallel y := y+2$ partition the variables:

put x in left part, put y and z in right part

$= \quad x' = x+1 \parallel y' = y+2 \land z' = z$

$= \quad x' = x+1 \land y' = y+2 \land z' = z$

reasonable partition rule

If either x' or $x:=$ appears in a process specification, then x belongs to that process

(then neither x' nor $x:=$ can appear in the other process specification).

If neither x' nor $x:=$ appears at all, then x can be placed on either side of the partition.

Independent Composition

example in variables x , y , and z

$x := y \,||\, y := x$ partition: put x in left, y in right, z in either

$=$ $x'=y \land y'=x \land z'=z$

implementation of a process makes a private copy of the initial value of a variable belonging to the other process if the other process contains an assignment to that variable

Independent Composition

example in boolean variable b and integer variable x

$b{:=}\ x{=}x \ \| \ x{:=}\ x{+}1$ replace $x{=}x$ by $\top$

$=$ $b{:=}\ \top \ \| \ x{:=}\ x{+}1$

example in integer variables x and y

$(x{:=}\ x{+}1.\ \ x{:=}\ x{-}1)\ \|\ y{:=}\ x$

$=$ $ok \,\|\, y{:=}\ x$

$=$ $y{:=}\ x$

Independent Composition

$(x{:=}\ x{+}y.\ \ x{:=}\ x{\times}y)\ \ \|\ \ (y{:=}\ x{-}y.\ \ y{:=}\ x/y)$

Independent Composition

$(x{:=}\ x{+}y.\ \ x{:=}\ x{\times}y) \ \ \| \ \ (y{:=}\ x{-}y.\ \ y{:=}\ x/y)$

You should have written

$(x{:=}\ x{+}y \ \| \ y{:=}\ x{-}y).\ \ (x{:=}\ x{\times}y \ \| \ y{:=}\ x/y)$

Independent Composition

$$P\|Q \;=\; \exists tP, tQ\cdot \quad (\text{substitute } tP \text{ for } t' \text{ in } P)$$
$$\wedge\; (\text{substitute } tQ \text{ for } t' \text{ in } Q)$$
$$\wedge\; t' = max\ tP\ tQ$$

laws

$(x{:=}\,e \,\|\, y{:=}\,f).\ P \;=\;$ (for x substitute e and independently for y substitute f in P)

$P \,\|\, Q \;=\; Q \,\|\, P$ — symmetry

$P \,\|\, (Q \,\|\, R) \;=\; (P \,\|\, Q) \,\|\, R$ — associativity

$P \,\|\, ok \;=\; ok \,\|\, P \;=\; P$ — identity

$P \,\|\, Q \vee R \;=\; (P \,\|\, Q) \vee (P \,\|\, R)$ — distributivity

$P \,\|\,$ **if** b **then** Q **else** R $=$ **if** b **then** $(P \,\|\, Q)$ **else** $(P \,\|\, R)$ — distributivity

if b **then** $(P\|Q)$ **else** $(R\|S)$ $=$ **if** b **then** P **else** R $\|$ **if** b **then** Q **else** S — distributivity

List Concurrency

$$Li:= e \quad = \quad L'i=e \ \wedge \ (\forall j\colon 0,..\#L\cdot\ j \neq i \Rightarrow L'j=Lj) \ \wedge \ x'=x \ \wedge \ y'=y \ \wedge \ ...$$

$$Li:= e \quad = \quad L'i=e \ \wedge \ (\forall j\colon \text{(this part)}\cdot\ j \neq i \Rightarrow L'j=Lj) \ \wedge \ x'=x \ \wedge \ ...$$

example find the maximum item in a nonempty list

$findmax\ 0\ (\#L)$ where

$$findmax \ = \ \langle i, j \rightarrow i<j \Rightarrow L'\ i = MAX\ L\ [i;..j] \rangle$$

$$findmax\ i\ j \ \Leftarrow \quad \textbf{if}\ j–i = 1\ \textbf{then}\ ok$$

$$\textbf{else}\ (\ (findmax\ i\ (div\ (i+j)\ 2)\ \|\ findmax\ (div\ (i+j)\ 2)\ j).$$

$$L\ i := max\ (L\ i)\ (L\ (div\ (i+j)\ 2))\ )$$

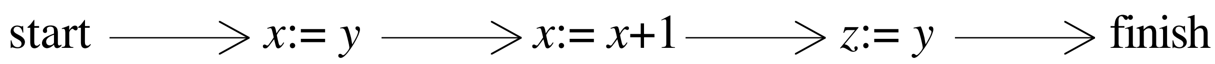
recursive time $=$ $ceil\ (log\ (j–i))$

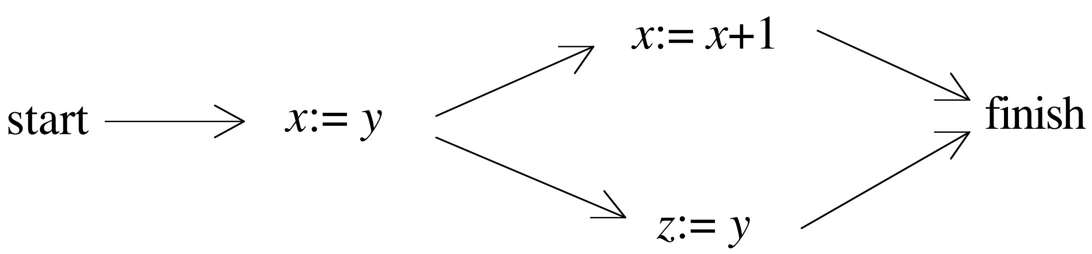
Sequential to Parallel Transformation

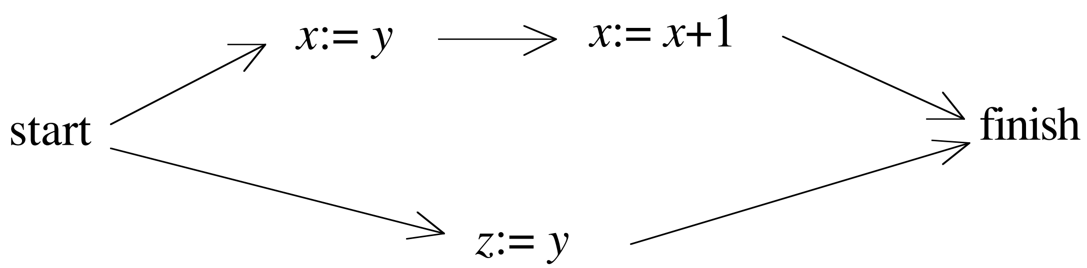
$$x := y.\ x := x+1.\ z := y$$

$$= \quad x := y.\ (x := x+1 \ \| \ z := y)$$

$$= \quad (x := y.\ x := x+1) \ \| \ z := y$$







Sequential to Parallel Transformation

rules

Whenever two programs occur in sequence, and neither assigns to a variable appearing in the other, they can be placed in parallel.

example $x{:=}\ z.\ \ y{:=}\ z$ becomes $x{:=}\ z \parallel y{:=}\ z$

Whenever two programs occur in sequence, and neither assigns to a variable assigned in the other, and no variable assigned in the first appears in the second, they can be placed in parallel; a copy must be made of the initial value of any variable appearing in the first and assigned in the second.

example $x{:=}\ y.\ \ y{:=}\ z$ becomes $c{:=}\ y.\ \ (x{:=}\ c \parallel y{:=}\ z)$

Buffer

$$produce = \cdots\cdots\cdots b := e \cdots\cdots\cdots$$

$$consume = \cdots\cdots\cdots x := b \cdots\cdots\cdots$$

$$control = produce.\ consume.\ control$$

$$P \longrightarrow C \longrightarrow P \longrightarrow C \longrightarrow P \longrightarrow C \longrightarrow P \longrightarrow C \longrightarrow$$

Buffer

$$produce = \cdots\cdots\cdots b := e \cdots\cdots\cdots$$

$$consume = \cdots\cdots\cdots x := b \cdots\cdots\cdots$$

$$control = produce.\ newcontrol$$

$$newcontrol = consume.\ produce.\ newcontrol$$

Buffer

$$produce = \cdots\cdots\cdots b := e \cdots\cdots\cdots$$

$$consume = \cdots\cdots\cdots x := b \cdots\cdots\cdots$$

$$control = produce.\ newcontrol$$

$$newcontrol = (consume \parallel produce).\ newcontrol$$

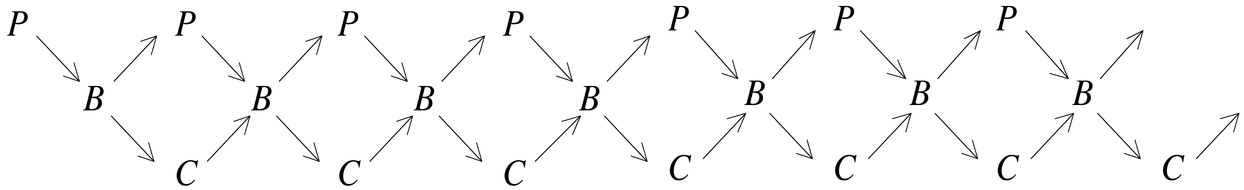
Buffer

$$produce = \cdots\cdots b := e \cdots\cdots$$

$$consume = \cdots\cdots x := c \cdots\cdots$$

$$control = produce.\ newcontrol$$

$$newcontrol = c := b.\ (consume \parallel produce).\ newcontrol$$



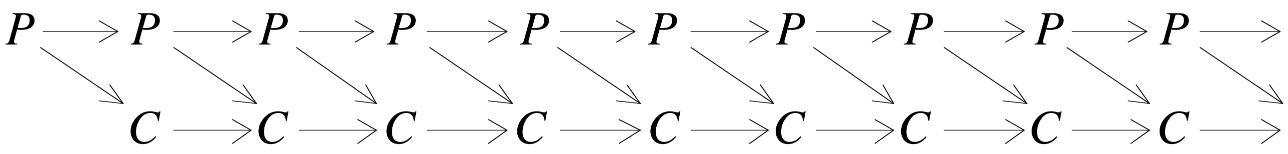
Buffer

$produce \; = \; \cdots\cdots b\, w := e.\;\; w := w+1 \cdots\cdots$

$consume \; = \; \cdots\cdots x := b\, r.\;\; r := r+1 \cdots\cdots$

$control \; = \; w := 0.\;\; r := 0.\;\; newcontrol$

$newcontrol \; = \; produce.\;\; consume.\;\; newcontrol$



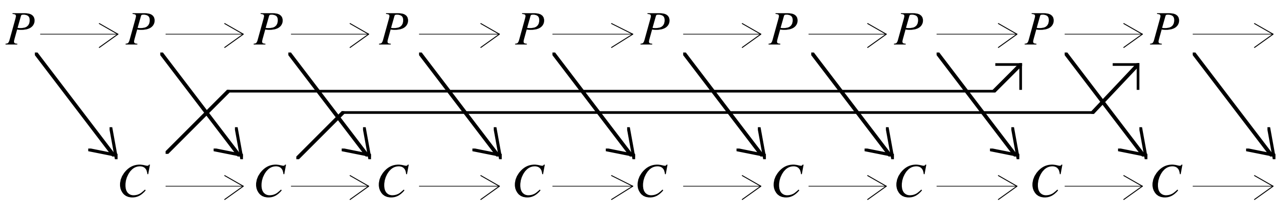
Buffer

$$produce \;=\; \cdots\cdots b\; w{:=}\,e.\;\; w{:=}\,mod\,(w{+}1)\,n\cdots\cdots$$

$$consume \;=\; \cdots\cdots x{:=}\,b\;r.\;\; r{:=}\,mod\,(r{+}1)\,n\cdots\cdots$$

$$control \;=\; w{:=}\,0.\;\; r{:=}\,0.\;\; newcontrol$$

$$newcontrol \;=\; produce.\;\; consume.\;\; newcontrol$$



Insertion Sort

define

$sort = \langle n \rightarrow \forall i, j{:}\ 0,..n\cdot\ i{\le}j \Rightarrow L\ i \le L\ j\rangle$

$swap = \langle i, j{:}\ 0,..\#L \rightarrow L\ i{:=}\ L\ j \parallel L\ j{:=}\ L\ i\rangle$

$sort'\ (\#L) \ \Leftarrow\ sort\ 0 \Rightarrow sort'\ (\#L)$

$sort\ 0 \Rightarrow sort'\ (\#L) \ \Leftarrow\ \textbf{for}\ n{:=}\ 0;..\#L\ \textbf{do}\ sort\ n \Rightarrow sort'\ (n{+}1)$

$sort\ n \Rightarrow sort'\ (n{+}1) \ \Leftarrow$

if $n{=}0$ **then** ok

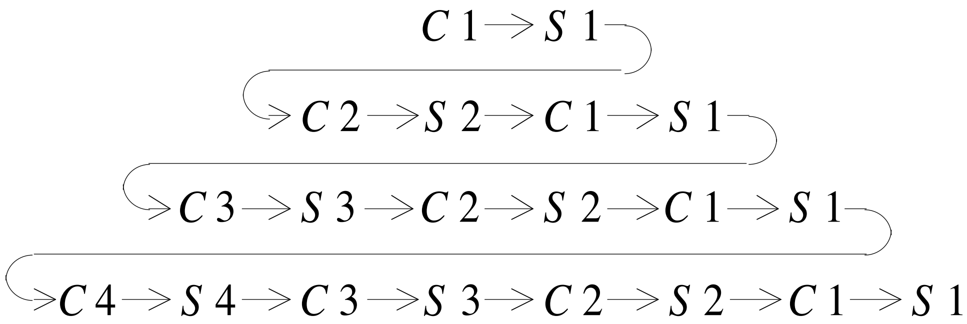
else if $L\ (n{-}1) \le L\ n$ **then** ok

else $(swap\ (n{-}1)\ n.\ \ sort\ (n{-}1) \Rightarrow sort'\ n)$

[$L\ 0$; $L\ 1$; $L\ 2$; $L\ 3$; $L\ 4$]

0 1 2 3 4 5

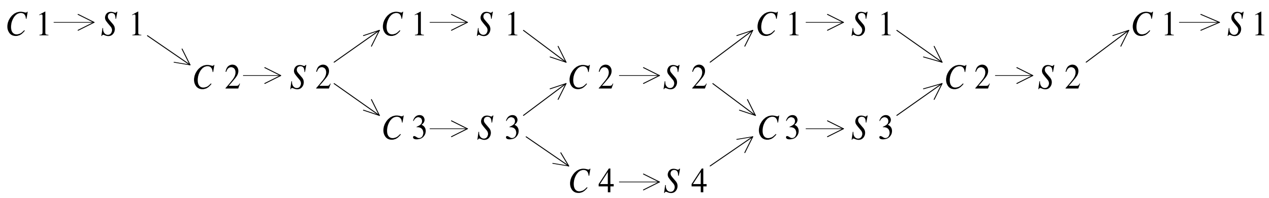
Insertion Sort



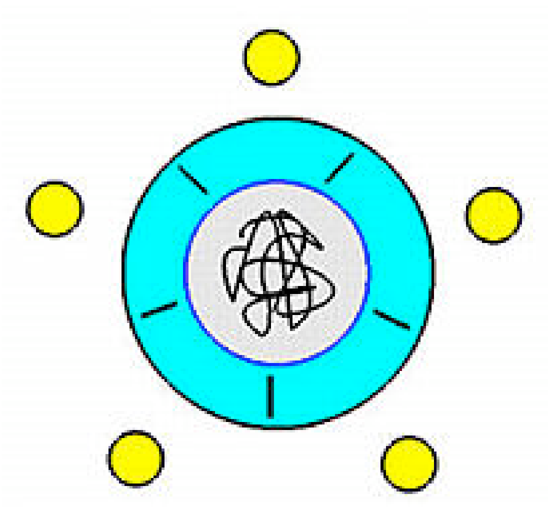
If *abs* $(i–j) > 1$ then *S i* and *S j* in parallel

If *abs* $(i–j) > 1$ then *S i* and *C j* in parallel

C i and *C j* in parallel



Dining Philosophers



Dining Philosophers

$$life = (P\,0 \lor P\,1 \lor P\,2 \lor P\,3 \lor P\,4).\ life$$

$$P\,i = up\ i.\ up(i+1).\ eat\ i.\ down\ i.\ down(i+1)$$

$$up\ i = chopstick\ i := \top$$

$$down\ i = chopstick\ i := \bot$$

$$eat\ i = \cdots\cdots chopstick\ i \cdots\cdots chopstick(i+1)\cdots\cdots$$

If $i \ne j$, $(up\ i.\ up\ j)$ becomes $(up\ i \parallel up\ j)$.

If $i \ne j$, $(up\ i.\ down\ j)$ becomes $(up\ i \parallel down\ j)$.

If $i \ne j$, $(down\ i.\ up\ j)$ becomes $(down\ i \parallel up\ j)$.

If $i \ne j$, $(down\ i.\ down\ j)$ becomes $(down\ i \parallel down\ j)$.

If $i \ne j \land i+1 \ne j$, $(eat\ i.\ up\ j)$ becomes $(eat\ i \parallel up\ j)$.

If $i \ne j \land i \ne j+1$, $(up\ i.\ eat\ j)$ becomes $(up\ i \parallel eat\ j)$.

If $i \ne j \land i+1 \ne j$, $(eat\ i.\ down\ j)$ becomes $(eat\ i \parallel down\ j)$.

If $i \ne j \land i \ne j+1$, $(down\ i.\ eat\ j)$ becomes $(down\ i \parallel eat\ j)$.

If $i \ne j \land i+1 \ne j \land i \ne j+1$, $(eat\ i.\ eat\ j)$ becomes $(eat\ i \parallel eat\ j)$.

Dining Philosophers

$$life \;=\; (P\,0 \lor P\,1 \lor P\,2 \lor P\,3 \lor P\,4).\;\; life$$

$$P\,i \;=\; up\ i.\;\; up(i{+}1).\;\; eat\ i.\;\; down\ i.\;\; down(i{+}1)$$

$$up\ i \;=\; chopstick\ i{:=}\top$$

$$down\ i \;=\; chopstick\ i{:=}\bot$$

$$eat\ i \;=\; \cdots\cdots chopstick\ i\cdots\cdots chopstick(i{+}1)\cdots\cdots$$

$$life \;=\; P\,0 \parallel P\,1 \parallel P\,2 \parallel P\,3 \parallel P\,4$$

$$P\,i \;=\; (up\ i \parallel up(i{+}1)).\;\; eat\ i.\;\; (down\ i \parallel down(i{+}1)).\;\; P\,i$$