Advanced Computer Architecture

Lecture No. 34

Reading Material

Vincent P. Heuring & Harry F. Jordan — Chapter 6
Computer Systems Design and Architecture — 6.1, 6.2

Summary

- Introduction to ALSU
- Radix Conversion
- Fixed Point Numbers
- Representation of Numbers
- Multiplication and Division using Shift Operation
- Unsigned Addition Operation

Introduction to ALSU [29]

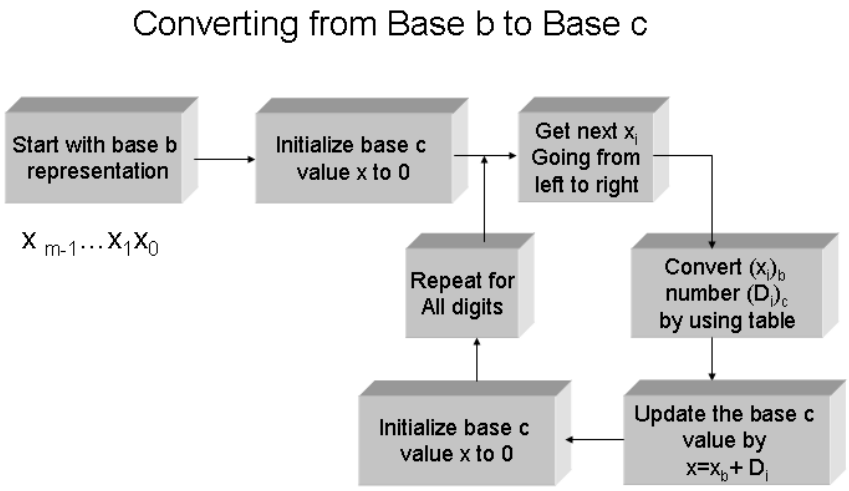
ALSU is a combinational circuit so inside an ALSU, we have AND, OR, NOT and other different gates combined together in different ways to perform addition, subtraction, and, or, not, etc. Up till now, we consider ALSU as a "black box" which takes two operands, a and b, at the input and has c at the output. Control signals whose values depend upon the opcode of an instruction were associated with this black box.

In order to understand the operation of the ALSU, we need to understand the basis of the representation of the numbers. For example, a designer needs to specify how many bits are required for the source operands and how many will be needed for the destination operand after an operation to avoid overflow and truncation.

Radix Conversion

Now we will consider the conversion of numbers from a representation in one base to another. As human works with base 10 and computers with base 2, this radix conversion operation is important to discuss here. We will use base c notion for decimal representation and base b for any other base. The following figure shows the algorithm of converting from base b to base c:

[29] In our discussion we have used ALU and ALSU for the same thing. We use ALSU when the shift aspect also needs to be emphasized.



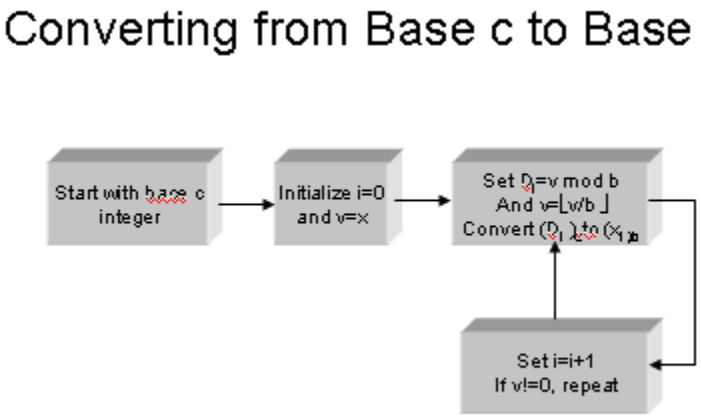
Example 1

Convert the hexadecimal number $B3_{16}$ to base 10.

Solution

According to the above algorithm,
X=0
X= x+B (=11) =11
X=16*11+3= 179
Hence $B3_{16}=179_{10}$

The following figure shows the algorithm of converting from base c to base b:



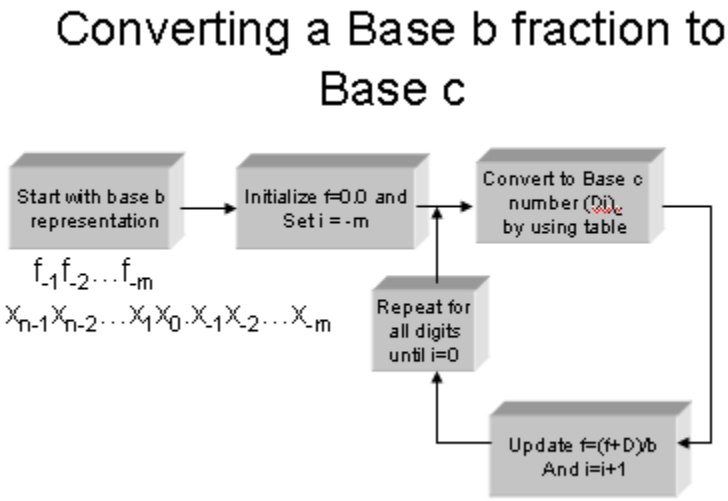
Example 2

Convert 390_{10} to base 16.
Solution

According to the above algorithm
390/16 =24(rem=6), x0=6
24/16= 1(rem=8), x1=8, x2=1
Thus $390_{10}=186_{16}$

Fixed Point Numbers

Suppose we have a number with a radix point. For example, in 16.12, there are two digits on the left side and two digits on the right of the decimal point. In this case, the radix point is a decimal point because we suppose that given number is a decimal number.
If we have an integer, then this decimal point will be on the right most position i.e. 1612.0 and if it is in fraction then decimal will be at the left most position i.e. 0.1612 There are situations when we shift the position of the radix point. Shifting of the radix point towards left or right is called scaling and we could have multiplication with a base or division by a base respectively.
The following figure shows the algorithm of converting a base b fraction to base c:



Example 3

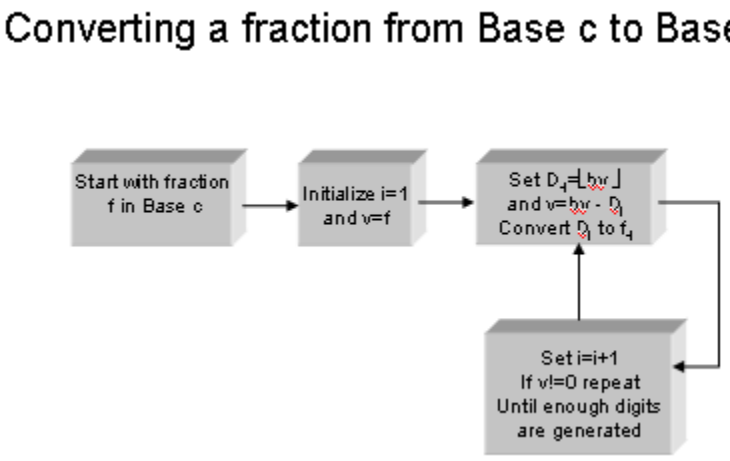
Convert $(.4cd)_{16}$ to Base 10.

Solution

F=0
F=(0+13)/16=0.8125

F=(0.8125+12)/16=0.80078125
F=(0.80078125+4)/16=$(0.3000488)_{10}$

The following figure shows the algorithm of converting fraction from base c to base b:



Example 4

Convert 0.24_{10} to base 2.

Solution

0.24*2=0.48, $f_{-1}=0$
0.48*2=0.96, $f_{-2}=0$
0.96*2=1.92, $f_{-3}=1$
0.92*2=1.84, $f_{-4}=1$
0.84*2=1.68, $f_{-5}=1$,…
Thus $0.24_{10}=(0.00111)_2$

Representation of Numbers

There are four possibilities to represent integers.

1. Sign magnitude form
2. Radix complement form
3. Diminished radix complement form
4. Biased representation

Sign magnitude form

- This is the simplest form for representing a signed number
- A symbol representing the sign of the number is appended to the left of the number

- This representation complicates the arithmetic operations

Radix complement form

- This is the most common representation.
- Given an m-digit base b number x, the radix complement of x is
  $x^c = (b^m – x) \bmod b^m$
- This representation makes the arithmetic operations much easier.

Diminished radix complement form

- The diminished radix complement of an m-digit number x is
  $x^{c'}=b^m -1- x$
- This complement is easier to compute than the radix complement.
- The two complement operations are interconvertible, as
  $x^c= (x^{c'}+1) \bmod b^m$

Table 6.1 of the text book shows the complement representation of negative numbers for radix complement and diminished radix complement form:
Table 6.2 of the text book shows the base 2 complement representation for 8-bit 2's and 1's complement numbers.

Example 5
The following table shows the decimal values in 2's complement, 1's complement, sign magnitude, 16's complement and in unsigned form:

Decimal	2's complement	1's complement	Sign-magnitude	16's complement	Unsigned
27	011011	011011	011011	1B	11011
.17	0.00101011	0.00101011	0.00101011	0.2B	0.00101011
-26	100110	100101	111010	E6	-
-0.57	1.01101110	1.01101101	1.10010010	F.6E	-

Multiplication and Division using Shift Operation

Shift left and shift right are two important operations used for various purposes. One typical example could be multiplication or division by base b. The following examples explain multiplication and division by using shift operation.

Example 6

- 6x4
  $00110_2 \times 4_{10} = 11000_2 = 24_{10}$

Overflow would occur if we will use 4 bits instead of 5 bits here.

- 60/16
  $0111100_2 / 16_{10} = 0000011_2 = 3_{10}$

The fractional portion of the result is lost.

Example 7

- -6x4
  $-6 = (11010)_2$
  $-6x4 = (01000)_2 = 8$ which is wrong!
  using less no. of bits might change sign

So, $-6 = (111010)_2$

$-6x4 = (101000)_2 = -24$

Example 8

Multiplication and division of negative numbers

Solution

-24x2

$-24 = (101000)_2$

$-24x2 = (010100)_2 = 20$

$-24x2 = (110100)_2 = -12$

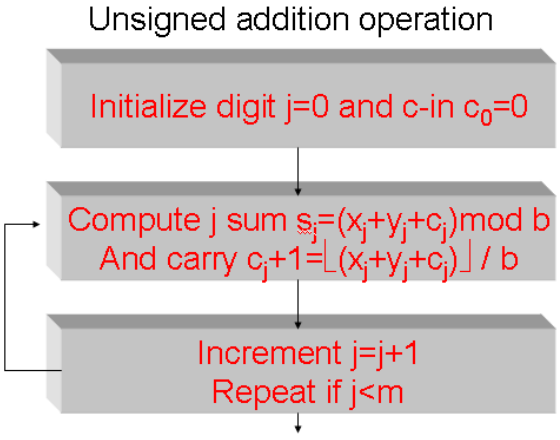
Changing the size of the number,

24= 011000 (n=6) to 00011000 (n=8)

-24= 101000 (n=6) to 11101000 (n=8)

Unsigned Addition Operation

The following diagram shows the digit wise procedure for adding m-digit base b numbers, x and y:



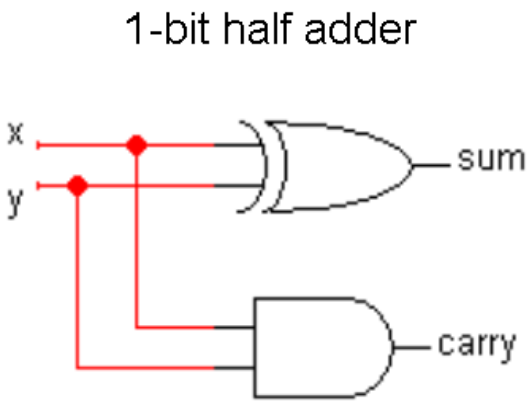
Example 9

Unsigned addition in base 2 and base16.

Solution

Base 16 addition	**Base 2 addition**
A B 4 2 $_{16}$ + 3 1 C 1 $_{16}$ carry 0 1 0 0 sum D D 0 3 $_{16}$	100011 $_{2}$ + 011011 $_{2}$ carry 000110 sum 111110 $_{2}$

The following diagram shows the logic circuit for 1-bit half adder. It takes two 1-bit inputs x and y and as a result, we get a 1-bit sum and a 1-bit carry. This circuit is called a half adder because it does not take care of input carry. In order to take into account the effect of the input carry, a 1-bit full adder is used which is also shown in the figure. We can add two m-bit numbers by using a circuit which is made by cascading m 1-bit full adders.



The situation, when addition of unsigned m-bit numbers results in an m+1 bit number, is called overflow. Overflow is treated as exception in some processors and the overflow flag is used to record the status of the result.