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SOME CHANGE OF VARIABLE FORMULAS IN INTEGRAL REPRESENTATION THEORY

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Abstract. Let X, Y be Banach spaces and let us denote by $C(S,X)$ the space of all X-valued continuous functions on the compact Hausdorff space S, equipped with the uniform norm. We shall write $C(S,X) = C(S)$ if $X = \mathbb{R}$ or $\mathbb{C}$. Now, consider a bounded linear operator $T : C(S,X) \to Y$ and assume that, due to the effect of a change of variable performed by a bounded operator $V : C(S,X) \to C(S)$, the operator T takes the product form $T = \theta \cdot V$, with $\theta : C(S) \to Y$ linear and bounded. In this paper, we prove some integral formulas giving the representing measure of the operator T, which appeared as an essential object in integral representation theory. This is made by means of the representing measure of the operator θ which is generally easier. Essentially the estimations are of the Radon-Nikodym type and precise formulas are stated for weakly compact and nuclear operators.

1. Introduction

Let S be a compact Hausdorff space and $\mathcal{B}_S$ the σ-field of the Borel sets of S. In all what follows, X and Y will be fixed Banach spaces and we consider the Banach space $C(S,X)$ of all X-valued continuous functions on S, with the uniform norm; we write $C(S,X) = C(S)$ when $X = \mathbb{R}$ or $\mathbb{C}$. In this work, we will be concerned with the integral analysis of bounded operators $T : C(S,X) \to Y$, taking the form:

$$T = \theta \cdot V \tag{1.1}$$

due to the effect of a change of variable performed by a bounded operator $V : C(S,X) \to C(S)$; θ being a bounded operator on $C(S)$ with values into Y. When the operators T and V are given, we will show how to get the operator $\theta : C(S) \to Y$, satisfying the product form (1.1). Then we determine the structure of the additive operator valued measure $G : \mathcal{B}_S \to \mathcal{L}(X, Y^{**})$ attached to the operator T via the integral representation:

$$f \in C(S,X), \quad Tf = \int_S f\, dG. \tag{1.2}$$

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